Prof. dr hab. inż. Ryszard Bartnik,
Politechnika Opolska, Wydział Inżynierii Produkcji i Logistyki,
Katedra Zarządzania Energetyką

mgr Berenika Bartnik,
absolwentka SGH

dr Anna Duczkowska-Kądziel
Politechnika Opolska, Wydział Inżynierii Produkcji i Logistyki,
Katedra Innowacyjnych Procesów Technologicznych

A formulate of problem of seeking an optimum investment strategy in power engineering

The power engineering sector in Poland, which is considerably decapitalized and technologically backward now faces considerable challenges. This statement is considerably confirmed by the fact that the necessary investment which is needed for modernizing the existing power units and construction of new ones amounting to at least 200 billion PLN. The need for involvement of such huge amounts requires that an optimum strategy of investing in the field should be found. However, it brings a need to search for an answer to the following questions: what technologies [2-6] should be applied for this purpose?; what effect on the final value of the adopted target criterion do the prices of energy carriers have and what are the relations between them?; how is it possible to phase the resources obtained from external sources over the years in order to achieve the assumed target in an assumed horizon? The following questions focus on the economic effectiveness of investment in the power sector. It is quite obvious that the economic result should be as high as possible while the costs associated with the generation of the electricity should be as low as possible [1]. Monograph [8] offers an interesting item which undertakes a thorough analysis of investments in the energy sector.

In order to analyze a given technical, economic phenomena or technical-economic ones, it is necessary to develop a mathematic model of it, i.e. establish a mathematical notation of the phenomenon which describes its function space. It is an all-time truth which states: *For the things of this world cannot be made known without a knowledge of mathematics*, as expressed in 13th century by Roger Bacon (1214-1294).

We sometimes face an opinion of economists that mathematical models which describe phenomena in economics consequently give it some weaknesses, or are totally unnecessary and even harmful. This claim is based on the statement that economics is a social and arts discipline. One cannot agree with any view of this kind. If a single mathematical model leads to errors and economic failure, this does not mean that modeling is generally wrong. It only indicates that a specific model was of poor quality. It also indicates that wrong assumptions were taken into account and a faulty theory which was taken as the basis for a model. The above principle is described by the GIGO (Garbage In, Garbage Out) phrase which says that if most nonsensical data (garbage in) is input into a system, the data will be produce nonsensical output (garbage out). If calculations using a faulty model apply even correct input data, the results from the calculations will be wrong as well. In the reverse case, if a model is correct and the data input into is nonsensical, the results will be wrong as well. In addition, if a model will be garbage quality, the output will be garbage as well. It is also possible to encounter a case when a phenomenon or variable is not taken into account while it should have been considered in a given model. Therefore, one needs to know that the final effect of any model cannot yield more information that is determined by the variables inherent as part of the modeled issue. Moreover, computer simulations gained a result of such modeling do not consider interactions caused by the unmodeled parameter. The function space will therefore not be true. For this reason it is necessary to bear in mind at all times that a model operates in a virtual space but does not analyze reality which is very complex and simply impossible to model by mathematical terms but it is only capable of analyzing a set of given assumptions which need to be constantly verified. In addition, it is necessary to continuously discuss and analyze the results of the performed calculations. It is also necessary to analyze the effect of the specific parameters on the final value of the analyzed one and how sensitive the latter are to the changing value of these parameters. It involves checking whether a small variation in the input in one parameter does not cause a considerable effect on the final calculations. As a result of these considerations, one can never speak of any final results provided by a mathematical model. One needs to bear in mind that one can only determine the probability of the occurrence of a modeled phenomenon and probable results based on this phenomenon.

A couple of words to add regarding the treatment of economics as a social science, i.e. economics as humanism. Such an approach equals economics as a voluble discipline, whereas important on one hand, like political sciences but leading only to bankruptcy and poverty and, hence, to destabilization and world destruction and even wars. One of the worst indication of treating economics as a social science, which is the role of demagogues and achieving particular benefits is associated with the will to print empty money, that is, money which does not have a cover in actual production. This is a way in which an economic catastrophe is delayed in time along with its consequences. In this manner a greater and greater sum of money is printed, while the effects are only faced by the citizens. The savings of the latter resulting from the long years of hard work are thus becoming worthless but only a part of trash money. As a consequence of this, prices of goods soar (one can note at this point that inflation is not associated with the increase of price but excess of money in the market whose result is visible in the rise of prices). One can additionally remark that in the economic life and beyond it, it is necessary to act responsibly and fairly but never be led by populism. One can never make a list of empty promise which give unattainable hopes. A tool for this purpose, except for *Decalogue* is offered by the development of mathematical models since *For the things of this world cannot be made known…* If these models are input the functions, in which one of the independent variables is time, then such models have the capability of predicting the future and results of calculations based on them will allow one to act rationally and responsibly. Obviously, models are not capable of mirroring the realty or the future, which is unknown (which is lucky since otherwise the world would go crazy and what would we do then?) but they allow us to perceive the inherent complex relations, which would otherwise be difficult or impossible to follow. In order to establish a correct mathematical model, it is necessary to form insightful and broad knowledge of the mind, which is based on history and psychology. This is so as a model should involve the sense of the profit thinking which is inherent in each human. It is natural for humans to have a enjoy a state of prosperity. This spirit has been living with us for centuries and does not change. It forms the basic motivation of the human action and is totally right. Only the scenery in which humans live change with time. This spirit has formed the most powerful force guiding progress and development as the prospect of prosperity forms the strongest drive in the search for new solutions and technologies developed by humans. This sense by its very nature occurs in mathematical models which are based on an economic criterion expressed by in the idea of the potential of gaining profits. It is a superior criterion and model in the technical solutions which are strived for. Technical analysis, though important and necessary is only capable of providing ways of improving technical and technology solutions and improving engineering solutions found in machines and facilities. However, finally, it is the economic criterion, one that is based on the maximization of profit, that is decisive in the justification and selection of a specific technical solution and decides about the particular actions which are taken on by humans.

In order to develop a correct mathematical model which accounts for the above described properties and characteristics, and one which is capable of predicting the future and, thus, enables one to follow the complex relations which occur as a result of business activities taken on by humans, it is necessary to have a genius or at least wisdom worth the Nobel Price. It is one of the most basic conditions in which correct results can be obtained from a model of mathematically modeled phenomenon. It is necessary to additionally have a sophisticated mathematical apparatus, which makes it possible to research and analyze a modeled function space. In this respect, it is particularly valuable to apply the theory of optimal strategies (systems) and in particular Pontryagin's maximum principle and Bellman principle of optimality. The first one find sits application in continuous processes while the latter to discreet ones.

Methodology of seeking optimum investment strategies in business

Functional analysis, i.e. a branch of mathematical analysis which deals with the study of characteristics of functional spaces, is the domain of research into mathematical models. The term *functional* denotes a function whose argument is a function and not a number. A specific branch of functional analysis is called calculus of variations, which deals with the search for extrema of functionals, which play an important role in the study of mathematical models.

A special case of finding extrema of functionals (i.e. *Mayer problem*) is the *Lagrange's problem*, which involves searching for extrema (maxima and minima) of integral functionals:

$$J = \int_{t_O}^{t_F} F\left[x_1(t), x_2(t), ..., x_n(t); u_1(t), u_2(t), ..., u_n(t); t\right] dt = \text{ekstremum}, \quad (1)$$

where:

$x_i(t)$ – dependent variables, i.e. state variables ($i = 1, 2, ..., n$); state variables are coordinates of the state vector $x(t) = [x_1(t), x_2(t), ..., x_n(t)]$,

t – independent variable (e.g. time), $t \in \langle t_O, t_F \rangle$,

whereas:

$$\frac{dx_i}{dt} \equiv u_i(t). \quad (2)$$

Lagrange's problem itself is a special case of the problem of *optimal solution* which involves determination of r of variable in the control $u_k = u_k(t)$ ($k = 1, 2, ..., r$) which give extrema of an integral functional (target criterion):

$$J = \int_{t_O}^{t_F} F\left[x_1(t), x_2(t), ..., x_n(t); u_1(t), u_2(t), ..., u_r(t); t\right] dt = \text{ekstremum}, \quad (3)$$

while the derivatives of function $x_i(t)$ fulfill in this case *n* number of differential equations of the first order called the state equations:

$$\frac{dx_i}{dt} = f_i\left[x_1(t), x_2(t), ..., x_n(t); u_1(t), u_2(t), ..., u_r(t); t\right], \quad (4)$$

while the controls $u_k(t)$ form the coordinates of the control vector $u(t) = [u_1(t), u_2(t), ..., u_r(t)]$.

Differential equations (4) describe the changes of process occurring in time described by means of a functional (3).

The optimal strategies, i.e. ones that give the extrema of a functional (3) the functions of control $u_k(t)$ determine the optimal trajectory $x_i = x_i(t)$ in an n dimensional state space.

In practice there is an extensive class of technical and economic solutions, in which in the place of a functional with a continuous time (3) and whose evolution is defined by differential equations (4), we have to do with processes which are discreet by their very nature. This class predominantly includes multistep tasks aimed at decision making. Economic processes are of this kind as it is always described by differential equations. The discretization step is determined by its cycle. In practice this is usually one year $\Delta t = 1$. In this case the search for an extremum of a target functional (target criteria) with the number of steps in a process to be included equal to N:

$$J = \sum_{t=1}^{N} F[x_1(t), x_2(t), ..., x_n(t); u_1(t), u_2(t), ..., u_r(t); t] = \text{ekstremum}, \quad (N = 1, 2, ...) \tag{5}$$

with differential state equations:

$$x_i(t-1) = f_i[x_1(t), x_2(t), ..., x_n(t); u_1(t), u_2(t), ..., u_r(t); t] \quad (i = 1, 2, ..., n) \tag{6}$$

it is possible to apply the *Bellman principle of optimality* [7]. This principle states that fact that all parts of an optimum trajectory optimizes a functional for the respective starting and final points. In other words, in order to ensure for a trajectory to be optimal, each of its parts (each step) needs to be optimal regardless of its initial points. This also means that the search for an optimum control needs to be performed for every step $t = 1, ..., N$ separately, with a respective initial point (extremum value) resulting from the preceding step, while each of the extrema has to be determined in accordance with the bond occurring in it. Thus, the Bellman principle allows one to search for an optimum of a functional by means of analyzing the extrema of a function and leads to a recurrent formula which expresses the total of N *Bellman equations*, in which letter S denotes the extrema of functions for steps „t – 1" and „t":

$$\begin{aligned} &S[x_1(t), x_2(t), ..., x_n(t); t] = \\ &= F[x_1(t), x_2(t), ..., x_n(t); u_1(t), u_2(t), ..., u_r(t); t] + \\ &+ S[x_1(t-1), x_2(t-1), ..., x_n(t-1); t-1], \qquad t = 1, ..., N \end{aligned} \tag{7}$$

For a uniform notation of formula (7) it was assumed that $S[x_1(0), x_2(0), ..., x_n(0); 0] = 0$.

Functions S in (7) are derived by means of applying optimum controls $u_k = u_k(t)$ (k = 1, 2, ..., r) calculated from the system of equations in (10).

The determination of a function equation (7) with respect to function $S[x_1(t), x_2(t), ..., x_n(t); t]$ is equivalent to the step-wise construction of a class of optimum strategies for a variety of initial states. This task for the number of relative variables equal to $x_i(t)$ above two becomes very extensive. Such lengths lead to the use of approximated methods when solutions of specific problems are sought. For less extensive problems the solution can be found more efficiently and faster by means of a method of approximations, even in the circumstances when high precision of calculations is required. In this case, the effective means involves the replacement of Bellman methodology (5) and differential state equations (6) with a continuous functional (3) and differential equations (4) and subsequent solution by Ritz method of approximation familiar from the calculus of variations or by application of the *method of Riccati substitution* [7].

Equations (6) and (7) present a normal (forward) recurrence. When in the search of an optimum trajectory the target is to reach the final point N with the value $x_i = x_i(N)$, the equations (6) and (7) needs to be substituted by backward recurrence:

$$x_i(t+1) = f_i[x_1(t), x_2(t), ..., x_n(t); u_1(t), u_2(t), ..., u_r(t); t] \tag{8}$$

$$\begin{aligned} &S[x_1(t), x_2(t), ..., x_n(t); t] \\ &= F[x_1(t), x_2(t), ..., x_n(t); u_1(t), u_2(t), ..., u_r(t); t] + \\ &+ S[x_1(t+1), x_2(t+1), ..., x_n(t+1); t+1], \qquad t = N-1, ..., 0 \end{aligned} \tag{9}$$

The backward recurrence, which by its nature is concerned with the backward logic of reasoning is a way of analyzing the future. It creates scientific thinking in the direction of the future. It starts with an adoption of a desired value in a year N and, subsequently, step by step, aims to model in the backward direction in order to obtain a present value which should ensure that the desired value can be achieved in year N. This backward thinking follows an optimal trajectory for an adoption target criterion and various alternatives of a control scenario, i.e. time variable prices of energy carriers, specific rates associated with emission of pollutants into the environment, etc. The calculated present value therefore indicates what technologies need to be adopted right now and imposes specific solutions needed in order to ensure that an optimum trajectory can lead to the desired target in the future. Therefore, it enables the analysis and selection of alternative solutions along with their inherent determinants in order to achieve a desired target value within a time horizon of N years. Therefore, it enables an analysis of variety of investment alternatives and technologies to be undertaken for a number of scenarios involving energy carrier prices and environmental considerations.

A noted above, the search for an optimum trajectory for a given target criterion (5) is associated with finding a solution (for each of successive steps t) of a system of r function equations in order to derive the r number of controls $u_k = u_k(t)$ (k = 1, 2, ..., r) which give the extremum in a given step t for a functional (5). This system can be presented in the following form:

$$\begin{cases} \dfrac{\partial\{F[x_1(t), x_2(t), ..., x_n(t); u_1(t), u_2(t), ..., u_r(t); t] + S[x_1(t\mp 1), x_2(t\mp 1), ..., x_n(t\mp 1); t\mp 1]\}}{\partial u_1} = 0 \\ \dfrac{\partial\{F[x_1(t), x_2(t), ..., x_n(t); u_1(t), u_2(t), ..., u_r(t); t] + S[x_1(t\mp 1), x_2(t\mp 1), ..., x_n(t\mp 1); t\mp 1]\}}{\partial u_2} = 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \dfrac{\partial\{F[x_1(t), x_2(t), ..., x_n(t); u_1(t), u_2(t), ..., u_r(t); t] + S[x_1(t\mp 1), x_2(t\mp 1), ..., x_n(t\mp 1); t\mp 1]\}}{\partial u_r} = 0 \end{cases} \tag{10}$$

in which the extremum of the function S has been determined in the preceding step t ∓ 1 (for the case of forward recurrence for the step t - 1, while in backward recurrence for the step t + 1).

In the system (10) prior to performing the operation of calculating partial differentials $\partial(F + S) / \partial u_k$ in the place of $x_i(t \mp 1)$ in the function S it is necessary to insert differential equations for the case of forward recurrence (6), and for the case of backward recurrence – equations (8).

Target functional in the search for an optimum investment strategy

During the search for an optimum investment strategy, the target criterion (optimality criterion) should involve the updated net present value (NPV), whose value is expressed by the formula [1]:

$$NPV = \sum_{t=1}^{N} \frac{S_{R,t} - K_{e,t} - F_t - R_t - (S_{R,t} - K_{e,t} - F_t - \frac{J_0}{N})p}{(1+r)^t} \rightarrow \max \tag{11}$$

which in continuous notation is expressed by the formula:

$$NPV = \int_{t_0}^{t_F} [S_R - K_e - F - R - (S_R - K_e - F - \frac{J_0}{N})p] e^{-rt} dt \rightarrow \max \tag{11a}$$

where:

F – time variable interest (financial cost) relative to investment J_0; interest is denoted by F and is an unknown function of the time variable installments R, $F = F(R)$,

J_0 – investment expenditure; it is relative to the adopted technology in power industry,

K_e – variable in time exploitation costs [1],

N – calculated exploitation time of an enterprise expressed in years,

p – variable in time rate of income tax,

R – variable in time installment connected with loan repayment,

r – variable in time discount rate,

S_R – variable in time annual turnover.

Whereas the total of installments for loan repayment R is imposed with a restriction. It has to be equal to the investment expenditure J_0. The relation of constraints is therefore expressed in the form:

$$\sum_{t=1}^{N} R_t = J_0 . \tag{12}$$

and in continuous notation

$$\int_{t_0}^{t_F} R dt = J_0 \tag{12a}$$

The investment expenditure J_0 is relative to the technology and relative to the electrical capacity of the power plant. Therefore, it assumes different value for the same value of electrical capacity depending on the adopted technology.

A selection of an investment strategy should be adopted for the goal of $NPV \rightarrow max$ for an adopted value of electrical capacity N_{el} of the power plant. The values which are optimized (variables which guide decision making) include:

- accessible technologies, and
- constutuent parts of this technology its technical solutions, including the use of specific equipment, its engineering parameters, rated capacities, structure of connections, exploitation parameters of the process, etc.

The value of the adopted electrical capacity N_{el} of the power plant in optimization calculations using the *Bellman principle of optimality* remains constant at all times. The investment associated with the construction of a power plant involves a long-year process which consists of multiple tasks: getting the building permission, gaining the sources of funding of the necessary investment, design stage process and construction period; therefore, the electrical capacity N_{el} has to be adopted at a constant level over the entire calculation period, i.e. over the total number of years $t = 1, 2, ..., N$. The time variables in this period include only the control variables of state, i.e. turnover and annual operating costs of the power plant (exploitation cost plus capital costs) which are additionally dependent on the adopted technology and planned electrical capacity N_{el}.

Below are presented state equations expressed by continuous notation for the processes described by the functional (11):

- state equation for the cost of finance:

$$\frac{dF}{dt} = -rR \tag{13}$$

where R is the control; for example for the differential notation the above state equation for the backward recurrence takes the form

$$F_{t+1} = F_t - rR_t , \qquad (\Delta t = 1\,\text{year}) \tag{13a}$$

- evolution of turnover $S_R = E_{el,R}e_{el}$ gained from the sales of electricity (for the case of combined cycle with concurrent electricity and heat production it will consist of the revenues gained from the sales of both electricity and heat, $S_R = E_{el,R}e_{el} + Q_R e_c$)

$$\frac{dS_R}{dt} = \frac{\partial S_R}{\partial e_{el}} \frac{de_{el}}{dt} = E_{el,R}\, a_{el} e_{el}^{t=0} e^{a_{el}t} \tag{14}$$

whereas the time variability of price of electricity e_{el} (specific per unit of energy) is described by the equation

$$e_{el} = e_{el}^{t=0} e^{a_{el}t} \tag{15}$$

- evolution of fuel cost $K_{pal} = E_{ch,R}e_{pal}$

$$\frac{dK_{pal}}{dt} = \frac{\partial K_{pal}}{\partial e_{pal}} \frac{de_{pal}}{dt} = E_{ch,R}\, a_{pal} e_{pal}^{t=0} e^{a_{pal}t} \tag{16}$$

whereas the time variability of price of fuel e_{pal} (specific per unit of energy) is described by the equation

$$e_{pal} = e_{pal}^{t=0} e^{a_{pal}t} \tag{17}$$

- evolution of the cost $K_{śr} = E_{ch,R}(\rho_{CO2}\, p_{CO2} + \rho_{CO}\, p_{CO} + \rho_{NOx}\, p_{NOx} + \rho_{SO2}\, p_{SO2} + \rho_{pył}\, p_{pył})$ associated with the use of the environment is expressed as

$$\begin{aligned} \frac{dK_{śr}}{dt} &= \frac{\partial K_{śr}}{\partial p_{CO_2}} \frac{dp_{CO_2}}{dt} + \frac{\partial K_{śr}}{\partial p_{CO}} \frac{dp_{CO}}{dt} + \\ &+ \frac{\partial K_{śr}}{\partial p_{NO_x}} \frac{dp_{NO_x}}{dt} + \frac{\partial K_{śr}}{\partial p_{SO_2}} \frac{dp_{SO_2}}{dt} + \frac{\partial K_{śr}}{\partial p_{pył}} \frac{dp_{pył}}{dt} = \\ &= E_{ch,R} (\rho_{co_2} a_{CO_2} p_{CO_2}^{t=0} e^{a_{CO_2}t} + \rho_{co} a_{CO} p_{CO}^{t=0} e^{a_{CO}t} + \\ &+ \rho_{NO_x} a_{NO_x} p_{NO_x}^{t=0} e^{a_{NO_x}t} + \rho_{SO_2} a_{SO_2} p_{SO_2}^{t=0} e^{a_{SO_2}t} + \rho_{pył} a_{pył} p_{pył}^{t=0} e^{a_{pył}t}) \end{aligned} \tag{18}$$

whereas the time variability of rates for CO_2, CO, NO_x, SO_2 and dust emission are defined by the equations

$$p_{CO_2} = p_{CO_2}^{t=0} e^{a_{CO_2}t} \tag{19}$$

$$p_{CO} = p_{CO}^{t=0} e^{a_{CO}t} \tag{20}$$

$$p_{NO_x} = p_{NO_x}^{t=0} e^{a_{NO_x}t} \tag{21}$$

$$p_{SO_2} = p_{SO_2}^{t=0} e^{a_{SO_2}t} \tag{22}$$

$$p_{pył} = p_{pył}^{t=0} e^{a_{pył} t} \tag{23}$$

- evolution of the cost of purchase $K_{CO2} = E_{ch,R}(1\text{-}u)\rho_{CO2}\, e_{CO2}$ of additional CO_2 emission allowances is expressed as

$$\frac{dK_{CO_2}}{dt} = \frac{\partial K_{CO_2}}{\partial e_{CO_2}} \frac{de_{CO_2}}{dt} = E_{ch,R}(1-u)\rho_{co_2} b_{CO_2} e_{CO_2}^{t=0} e^{b_{CO_2} t} \tag{24}$$

while the time variable prices e_{CO2} of purchasing additional CO_2 emission allowances (specific, per unit of mass) are described with the formula

$$e_{CO_2} = e_{CO_2}^{t=0} e^{b_{CO_2} t} \tag{25}$$

where:

a_{el}, a_{pal}, a_{CO2}, a_{CO}, a_{SO2}, a_{NOx}, $a_{pył}$, b_{CO2} – control variables,
$E_{el,R}$ – annual net production of electricity,
$E_{ch,R}$ – annual use of the chemical energy of fuel,
u – proportion of chemical energy of fuel in its total use, for which it is not necessary to purchase CO_2 emission allowances,
ρ_{CO2}, ρ_{CO}, ρ_{NOx}, ρ_{SO2}, $\rho_{pył}$ – emission of CO_2, CO, NO_x, SO_2, and dust per specific unit of chemical energy of the fuel.

In search for a maximum of the target functional (11), it is also possible to analyze the effect of the efficiency of the facilities applied in the particular technologies on its overall value. For this purpose, the state equation in variables $E_{el,R}$ and $E_{ch,R}$ should include energy balances developed for the examined technologies [2-6].

Conclusions

The application of specific technology and its solutions determine the amount that is needed for an investment J_0 connected with the construction of a power plant (in a general case this may be a source of electricity and heat). Hence, it decides on the value of the cost of finance F and credit installments R which determine the annual operating costs of a power plant $t = 1, 2, ..., N$ and along with the prices of energy carriers and specific rates for pollutant emissions into the environment decide on the annual turnover S_R and annual exploitation costs K_e. Therefore, the selection of technology decides on the value of NPV. An optimum strategy regarding the technology will therefore be associated with the selection of a single one for which the value of NPV along with the application of the *Bellman principle of optimality* gains a maximum value for an adopted values of the overall capacity N_{el} of a power plant.

At the same time one can note that investment decisions are long-term ones and inseparably are connected with a risk of failure. The assessment of time and its associated risk is difficult and sometimes impossible to predict, in particular in unstable economic situation. This risk, however, does not release an investor from the need of searching for an optimum investment strategy. The search for it can enable one to analyze the resulting description of the prospects. This can also provide means to analyze the future in a scientific manner, i.e. in a way that involves the considerations of variable price relations between energy carriers, costs associated with the use of the environment, all factors which can affect the decision regarding the selection of a strategy. The results of searching for a maximum of the functional (11) should therefore provide information how the price relation and environmental charges influence the selection of an investment strategy and optimum energy technology.

A means which can minimize the risk can be associated with diversification of the technology, which also involves the need to analyze the future prospects. In contrast, this can lead to a rational decision regarding diversification of the applied technologies to ensure that the most effective one from the economic perspective is selected. As a result, the security of the supply of electricity can be additionally improved.

In summary, the application of mathematical models in economics and their analysis by means of forward thinking scenarios will consequently enable the rational selection of a strategy to be adopted in order to ensure that the desired characteristics can be achieved in an optimum manner.

REFERENCES

[1] Bartnik R.: Rachunek efektywności techniczno-ekonomicznej w energetyce zawodowej, Oficyna Wydawnicza Politechniki Opolskiej, Opole 2008.

[2] Bartnik R.: Elektrownie i elektrociepłownie gazowo-parowe. Efektywność energetyczna i ekonomiczna, Wydawnictwa Naukowo-Techniczne, Warszawa 2009 (dodruk 2012).

[3] Bartnik R., Buryn Z.: Conversion of Coal-Fired Power Plants to Cogeneration and Combined-Cycle. Thermal and Economic Effectiveness, Wydawnictwo Springer-Verlag, London 2011.

[4] Bartnik R., Buryn Z., Duczkowska-Kądziel A.: Modernizować istniejące, czy budować nowe źródła energii elektrycznej? *Energetyka* 2012, nr 11.

[5] Bartnik R.: The Modernization Potential of Gas Turbines in the Coal-Fired Power Industry. Thermal and Economic Effectiveness, Wydawnictwo Springer-Verlag, London 2013.

[6] Chmielniak T.: Technologie energetyczne, Wydawnictwa Naukowo-Techniczne, Warszawa 2008.

[7] Findeinsen W., Szymanowski J., Wierzbicki A.: Teoria i metody obliczeniowe optymalizacji. PWN, Warszawa 1980.

[8] Sowiński J.: Inwestowanie w źródła wytwarzania energii elektrycznej w warunkach rynkowych. Wydawnictwo Politechniki Częstochowskiej, Częstochowa 2008.